

A Theoretical Approach to a pure Optical Navigation System

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ABSTRACT

State-of-the-art Inertial Navigation Systems (INS) based on Micro-Electro-Mechanical Systems (MEMS) have a lack of precision especially in GPS denied environments like urban canyons or in pure indoor missions. Recent technological advances in both optical flow field estimation and low-cost high performance DSP/FPGA processors enabled the feasibility of purely optical navigation. The proposed Optical Navigation System (ONS) consists of three cameras pointing to each body axes which implies a triple redundancy of the bias free ego-motion estimates. Simulating a high performance flow field estimator our system can compete with a conventional low-cost INS (e.g. equipped with an Analog Devices ADIS 16355 6 DoF inertial unit) when various object camera distances are present.

Keywords: Optical Navigation, Low-Cost, Optical Flow, Sensor-Fusion

1. INTRODUCTION

In today's technological applications Micro Electro Mechanical System (MEMS) based inertial sensors are often used, in order to sense the ego-motion of a body in the three-dimensional space. Due to the random walk behaviour of inertial sensors these are often combined with other types of sensors. For applications without any restrictions of the satellite connection an inertial navigation system (INS) in combination with the Global Positioning System (GPS) represents a state-of-the-art navigation solution.¹ Especially in areas where satellite signals are shaded due to structural conditions, a GPS/INS navigation solution loses its accuracy.²

In the past the focus lay on replacing the GPS information. Thus depth measurements were used on basis of ultrasonic or laser scanners. In addition map-based procedures were included to support the INS.

More than fifty years ago, Gibson pointed out that visual motion perception is essential for an observer to explore and interact with his/her environment.³ Since that time, perception of motion has been studied extensively by researchers in the field of computer vision.⁴⁻⁶ Nowadays it is well known that humans are capable of recovering accurate ego-motion information from relative motion in images. Therefore enhanced navigation solutions have been explored that process visual information.⁷⁻⁹

For applications which include in particular indoor operations and urban environments this article presents a new approach for a navigation system that is purely based on optical sensors. This optical navigation system (ONS) should be autonomous and may be used in unknown dynamic environments. Not depending on external sensor information our system is able to completely work autonomously in GPS denied environments.

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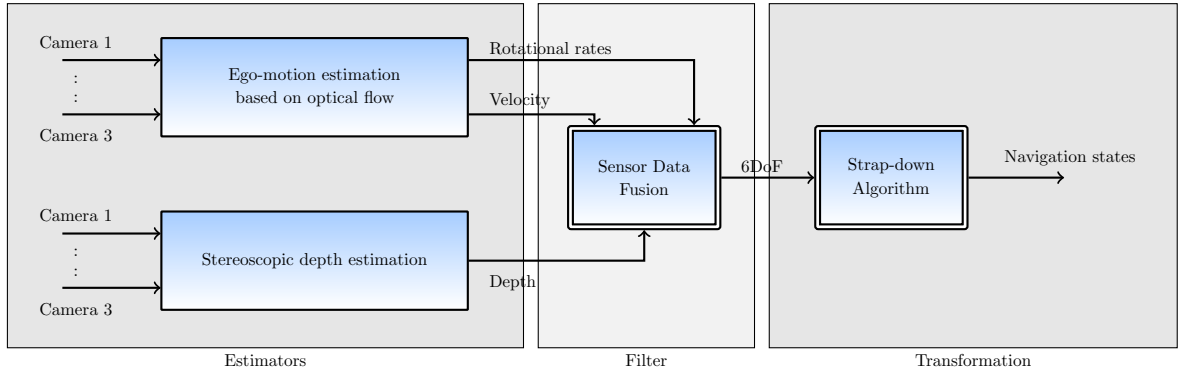


Figure 1. Conceptual proposal for an optical navigation system (ONS)

1.1 Conceptual Proposal for an Optical Navigation System

The concept for the ONS envisages the use of three monocular cameras pointing to each of the body axes. This arrangement offers three advantages: (i) The ONS is isotropic so that any movement in three-dimensional space is detected equally. (ii) High error tolerance regarding disturbances in one camera channel since the information is redundantly available. (iii) The redundant information is uncorrelated so that a failure of one camera channel does not affect the other two channels.

In figure 1 the concept proposal of such an optical navigation system is shown schematically. The proper motion is evaluated independently for each of the three cameras. The provided ego-estimation method evaluates the optical flow and is described in section 2. This method is mainly based on the approach of Raudies and Neumann in the year 2009.⁶ Their method features the estimation of all six degree of motion \mathbf{v}^c , $\boldsymbol{\omega}^c$ with a single monocular camera. A minor drawback of this method is that the estimated translational velocity vector $\hat{\mathbf{v}}^c$ can only be represented as a unity vector. The velocity of the body follows after multiplying by a constant κ with:

$$\hat{\mathbf{v}}^c = \kappa \hat{\mathbf{v}}^c \quad \text{with} \quad 0 \leq \kappa \leq \infty \quad (1)$$

Including a distance sensor it is possible to estimate the scale factor κ to eliminate this disadvantage.

As possible distance sensors ultrasonic or laser scanners could be used for this area of application.¹⁰ Continuing the purely optical approach a depth estimation via stereoscopy is conceivable.^{11,12} The stereoscopic distance measurement offers three advantages over ultrasonic or laser measurements (i) There are a variety of distance information for each measurement available so that a better estimation of the scaling factor κ is possible. (ii) In the case of use of a further camera an additional estimation of the independent movement can take place. (iii) It is conceivable to use a single camera for a stereoscopic measurement so that no further sensor system is needed.^{12,13} Due to these advantages our concept recommends the usage of a stereoscopic depth estimation.

While every camera measures all 6 degrees of freedom a triple redundancy is given and therefore a sensor data fusion filter is needed. In this case a linear Kalman filter was chosen. By prediction of the measured values an online covariance estimation is feasible and it is also possible to predict the integrity of the navigation solution. The proposed fusion filter is described in section 3. If the ego-motion is determined in body-fixed coordinates it can be transformed with a following strap-down algorithm into a navigation and/or earth coordinate system to provide a solution of the well-known navigation states *position*, *velocity* and *attitude*. The strap-down algorithm is presented in section 4.

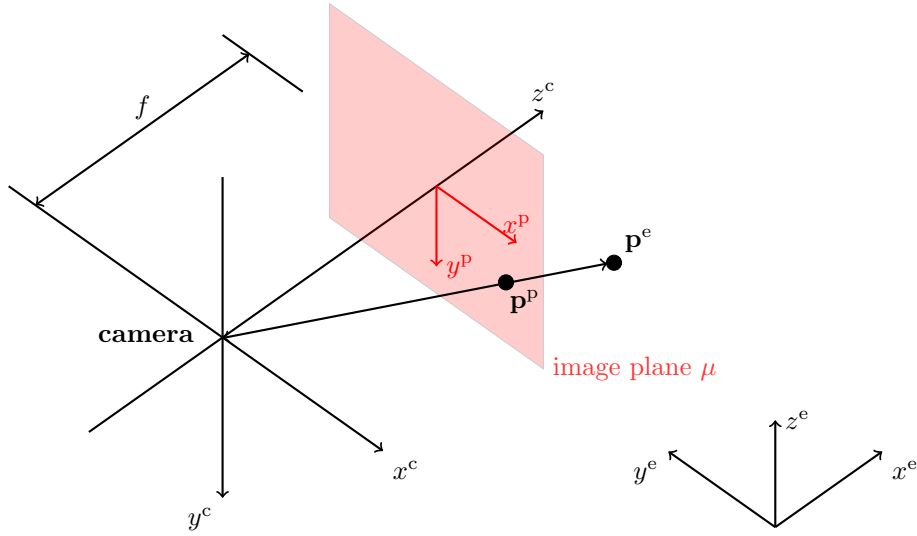


Figure 2. Projective geometry for a pinhole camera

2. EGO-MOTION ESTIMATION

As initially mentioned the approach of Raudies and Neumann is based on optical flow and provides an estimation of motion within all 6 degrees of freedom with a monocular camera. Their method relies on the instantaneous ego-motion model which is relating three-dimensional ego-motion to an optical flow field according to equation (2). This equation has been derived previously by a number of authors.^{4,5,14} In their description they used a pinhole camera with the focal length f which projects each three-dimensional point in a scene \mathbf{p}^e onto the two-dimensional image plane μ as depicted in figure 2.

$$\begin{bmatrix} \dot{x}^p \\ \dot{y}^p \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \mathbf{u} = \frac{1}{z^c} \mathbf{A} \mathbf{v}^c + \frac{1}{f} \mathbf{B} \boldsymbol{\omega}^c \quad (2)$$

with

$$\mathbf{A} = \begin{bmatrix} -f & 0 & x^p \\ 0 & -f & y^p \end{bmatrix} \quad (3)$$

$$\mathbf{B} = \begin{bmatrix} x^p y^p & -(f^2 + (x^p)^2) & f y^p \\ (f^2 + (x^p)^2) & -x^p y^p & -f x^p \end{bmatrix} \quad (4)$$

Equation (2) describes the flow field $\mathbf{u} = [u_x \ u_y]^T$ as a function of three-dimensional ego-motion (\mathbf{v}^c , $\boldsymbol{\omega}^c$) and depth z^c . An important observation about equation (2) is its bilinearity; \mathbf{u} is a linear function of \mathbf{v}^c and $\boldsymbol{\omega}^c$ for fixed z^c , and it is a linear function of z^c and $\boldsymbol{\omega}^c$ for fixed \mathbf{v}^c .

Since the depth z^c and the velocity \mathbf{v}^c are unknowns and since they are multiplied with each other in equation (2) they can each be determined only up to a scale factor κ . So it is only possible to solve the direction of velocity and the relative depth not for the absolute velocity nor the absolute depth. For the rest of this article $\bar{\mathbf{v}}^c$ denotes a unit vector in the translation direction (according to equation (1)).

Based on equation (2) there are numerous publications which allow an estimate of the proper motion.^{4,15,16} The special feature of Raudies and Neumann is the transfer into a linear optimization problem by introduction

of auxiliary variables. Their method is derived in three consecutive steps: (i) formulation of a linear, depth-independent optimization problem; (ii) estimating the translational velocity; (iii) removal of a statistical bias due to noisy flow vectors; (iv) estimating the rotational rates.⁶

Their method leads to the following linear, depth-independent optimization problem with nine equations and nine unknowns.

$$\mathbf{0} = (\bar{\mathbf{v}}^c)^T \int_{\Omega} \mathbf{L}_i \mathbf{L}_j \, d\mathbf{p}^P = (\bar{\mathbf{v}}^c)^T \mathbf{S} \quad \text{with} \quad i, j \in [1, 3] \quad (5)$$

with

$$\mathbf{L}_i = \mathbf{M}_i - (\mathbf{Q}\mathbf{E})^T \int_{\Omega} \mathbf{E}\mathbf{M}_i \, d\mathbf{p}^P \quad \text{with} \quad i, j \in [1, 3] \quad (6)$$

and

$$\mathbf{Q} = \left[\int_{\Omega} \mathbf{E}\mathbf{E}^T \, d\mathbf{p}^P \right]^{-1}, \quad \mathbf{M} = \begin{bmatrix} f u_x \\ -f u_y \\ y^P u_x - x^P u_y \end{bmatrix} \quad (7)$$

$$\mathbf{E} = \left[-\left(f^2 + (x^P)^2\right) \quad x^P y^P \quad f x^P \quad -\left(f^2 + (x^P)^2\right) \quad f y^P \quad -\left((x^P)^2 + (y^P)^2\right) \right]^T. \quad (8)$$

A robust (non-trivial) solution for such a system is given by the eigenvector which corresponds to the smallest eigenvalue of the 3×3 scatter matrix \mathbf{S} .⁴

As already mentioned above the solution space of the velocity vector $\bar{\mathbf{v}}^c$ describes a unit sphere in three dimensional space. By the expression $\mathbf{S} = \mathbf{L}_i \mathbf{L}_j$ the vectors are squared so that the resolution space reduces on a hemisphere.

This article extends the approach of Raudies and Neumann by estimating the sign of the velocity $\bar{\mathbf{v}}^c$ by a constant $\tilde{\kappa} \in \{-1, 1\}$. This leads to a optimization problem by minimizing the Euclidean distance of the residual between the estimated flow vector $\hat{\mathbf{u}} = [\hat{u}_x \hat{u}_y]^T$ from a spatio-temporal image sequence and the model flow vector \mathbf{u} defined in Equation (2).

$$R(\tilde{\kappa}) = \|\mathbf{u} - \hat{\mathbf{u}}(\tilde{\kappa})\|_2^2 \xrightarrow{\tilde{\kappa}} \min \quad \text{with} \quad \tilde{\kappa} \in \{-1, 1\} \quad (9)$$

3. SENSOR DATA FUSION

In general sensor data fusion provides techniques and tools which are used for combining sensor data or information derived from sensor data and can be divided into two areas of applications: (i) the fusion of multiple measurements produced at different time instants by a single sensor; (ii) the fusion of multiple measurements by multiple sensors. In performing sensor fusion the aim is to improve the quality of the information so that it is in some sense “better” than would be possible if the data sources were used individually.

Based on the approach of an optical navigation system, sensor data fusion means combining multiple estimated ego-motion to increase the integrity and continuity of the estimate. From the perspective of sensor data fusion “estimated ego-motion” is interpreted as a measure. For the rest of this section “measured ego-motion” denotes “estimated ego-motion” with the method described in section 2.

Due to the selected concept with three cameras which are pointing along each body axes as well as using the ego-motion estimation method shown in section 2 one receives a triple redundant measurements of all six degree

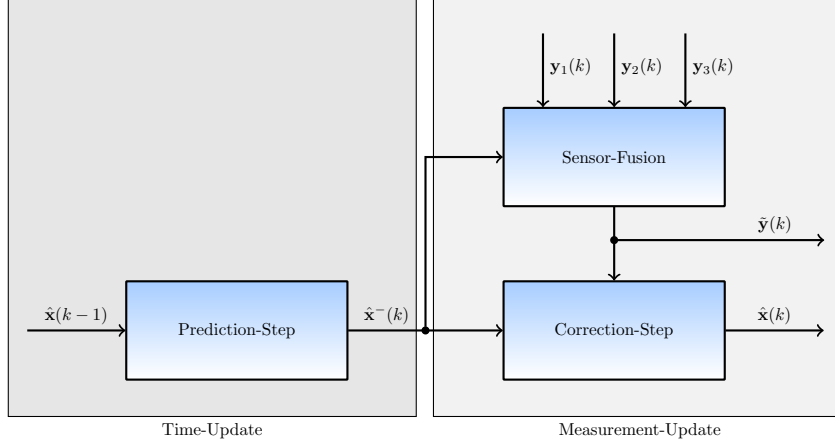


Figure 3. Block diagram of the used fusion algorithm

of freedom. Furthermore the measurements are linearly independent and uncorrelated since the coverage of each camera does not overlap. The aim of the sensor data fusion is an optimal blending of the measured information.

The optimal blending for three independent measurements is well-known:¹⁷ Let y_1 , y_2 and y_3 be three noisy measurements of the signal z

$$y_1(k) = z(k) + w_1(k) \quad y_2(k) = z(k) + w_2(k) \quad y_3(k) = z(k) + w_3(k) \quad (10)$$

with unbiased, uncorrelated and normally distributed noise with a standard deviation of σ

$$p(w_1(k) \sim N(0, \sigma_1)) \quad p(w_2(k) \sim N(0, \sigma_2)) \quad p(w_3(k) \sim N(0, \sigma_3)) \quad (11)$$

then the optimal blending leads to

$$\tilde{y}(k) = \frac{\sigma_2^2 \sigma_3^2 y_1(k) + \sigma_1^2 \sigma_3^2 y_2(k) + \sigma_1^2 \sigma_2^2 y_3(k)}{\sigma_1^2 \sigma_2^2 + \sigma_1^2 \sigma_3^2 + \sigma_2^2 \sigma_3^2}. \quad (12)$$

The necessary variances $\sigma_i^2(k)$ can be estimated on-line with the predictor-corrector structure by a Kalman filter.

$$\hat{\sigma}_i^2(k) = (y_i(k) - \hat{y}_i(k))^2 \quad \text{with} \quad i \in [1, 3] \quad (13)$$

In equation (13) $y_i(k)$ denotes the real measurement and $\hat{y}_i(k)$ the predicted measurement. In this case a linear Kalman filter with the state vector $\mathbf{x} = [\bar{v}_x \ \bar{v}_y \ \bar{v}_z \ \dot{\Phi} \ \dot{\Theta} \ \dot{\Psi}]^T$ and no input vector \mathbf{u} is used.

$$\underbrace{\begin{bmatrix} \hat{v}_x(k) \\ \hat{v}_y(k) \\ \hat{v}_z(k) \\ \hat{\Phi}(k) \\ \hat{\Theta}(k) \\ \hat{\Psi}(k) \end{bmatrix}}_{\hat{\mathbf{x}}(k)} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} \hat{v}_x(k-1) \\ \hat{v}_y(k-1) \\ \hat{v}_z(k-1) \\ \hat{\Phi}(k-1) \\ \hat{\Theta}(k-1) \\ \hat{\Psi}(k-1) \end{bmatrix}}_{\hat{\mathbf{x}}(k-1)} \quad (14)$$

As it is shown by the system matrix \mathbf{A} the states are estimated as random constants. This is a usual approach for estimating unknown states.¹⁷ Since all states are measured the observation model is given by

$$\hat{\mathbf{y}}(k) = \mathbf{C} \cdot \hat{\mathbf{x}}(k) \quad (15)$$

with the observation matrix \mathbf{C} as 6×6 unity matrix. According to the estimated variances the fused measurement $\tilde{y}(k)$ results from equation (12) and the variance of the fused measurement is determined by

$$\tilde{\sigma}^2 = \frac{\sigma_1^2 \sigma_2^2 \sigma_3^2}{\sigma_1^2 \sigma_2^2 + \sigma_1^2 \sigma_3^2 + \sigma_2^2 \sigma_3^2} \quad (16)$$

This approach offers two advantages: (i) Reduce the variance of the merged measurement; (ii) it is also possible to reduce the influence of measurement errors such as outliers. The structure of such a filter approach is depicted in figure 3.

4. STRAP-DOWN ALGORITHM

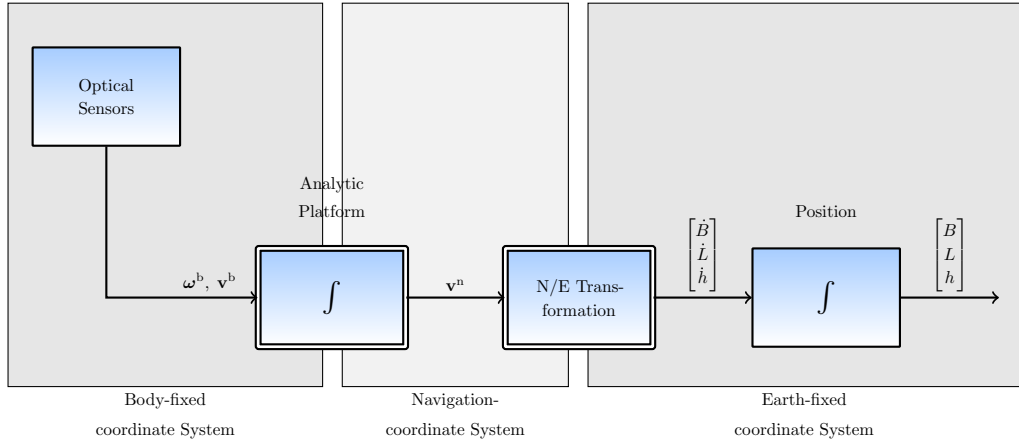


Figure 4. Simplified strap-down algorithm for the proposed optical navigation system

In general a strap-down algorithm calculates the well-known navigation solution latitude B , longitude L , height h , north-east-down velocity \mathbf{v}^n and attitude Φ , Θ , Ψ by integrating body-fixed inertial accelerations \mathbf{a}^b and rotational rates $\boldsymbol{\omega}^b$. Using inertial sensors also the earth rotation rate, earth gravity and coriolis forces are measured additionally to the independent movement of the body. With regard to the current position and velocity those influences have to be separated out later. Nevertheless small errors in position and velocity could already lead to substantial errors in the navigation solution.^{1,2} However, if optical sensors are used the strap-down algorithm can be substantially simplified because these inertial effects do not have to be considered. A simplified description of the strap-down algorithm is depicted in figure 4. Mainly the algorithm consists of an integration chain. On the basis of the initialized attitude the analytic platform integrates the body-fixed rotation rates $\boldsymbol{\omega}^b$ and determines the current attitude Φ , Θ , Ψ .

If the attitude is known the body-fixed velocity measurement \mathbf{v}^b can be transformed into the navigation coordinate system by a direction cosine matrix (DCM). Subsequently the velocity at the tangential plane is converted via the earth radius into angular velocities \dot{B} , \dot{L} and \dot{h} .²

5. RESULTS

The used test scenario was generated synthetically and simulates an indoor environment. For the analytical computation of the optical flow field equation (2) is used. The optical flow field exclusively results from the ego-motion of the observer and exhibits in particular independent moving objects (IMO). In the selected scenario the observer is moving along a given trajectory with a translational velocity $\mathbf{v}^b(t) = 0.3 \frac{\text{m}}{\text{s}}$ and rotational rates up to $200 \frac{\circ}{\text{s}}$. Thereby in particular also dynamic error influences are excited.

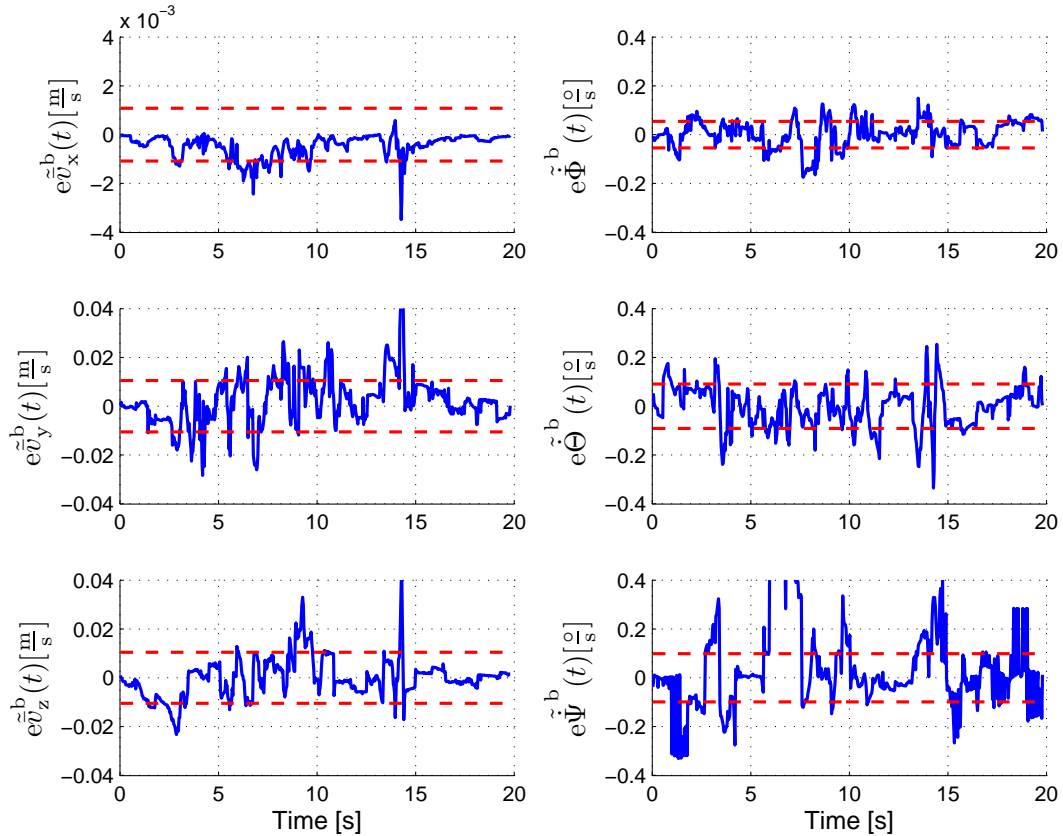


Figure 5. Errors of all six degree ego-motion estimates in body-fixed coordinates for one exemplary run. The red dashed line denotes the optimal variance.

Figure 5 shows the error of the fused measurement $\tilde{\mathbf{y}} = [\tilde{v}_x \ \tilde{v}_y \ \tilde{v}_z \ \tilde{\Phi} \ \tilde{\Theta} \ \tilde{\Psi}]^T$. The underlying optical flow field contains 10×10 vectors, as well as an additive white noise with the mean $\mu = 0$ and variance of $\sigma^2 = 10^{-2}$ pixels. In the selected scenario the velocity vector $\mathbf{v}^b(t)$ coincides with one of the camera axes. Therefore in all three cameras the flow is caused only by one velocity component (in this case along the body-fixed x axis). Due to the pure excitation in body-fixed x-direction the amounts of the corresponding flow vectors are very large in the comparison to the noise. This leads to a very good estimation of $\hat{v}_x(t)$ with a dispersion of $\leq 10^{-3} \frac{\text{m}}{\text{s}}$. In the two other body axes only a very small excitation takes place, thus only optical flow noise ($\mu = 0, \sigma^2 = 10^{-2}$ pixel) is present in the velocity components $\hat{v}_y(t), \hat{v}_z(t)$. Finally the statement of Raudies and Neumann of a bias free estimation can be confirmed on the assumption of a bias free flow field.⁶

A comparable statement can be given for the estimation of the rotational rates $\omega^b(t)$ which are also depicted in figure 5. The selected trajectory enables good excitations in all three body-fixed axes therefore a good estimation of the rotational rates is possible. The dispersion is approximately $0.1 \frac{\circ}{\text{s}}$ with 30 fps.

For the usually small operational areas of indoor applications a Cartesian coordinate system is sufficient. Therefore the transformation between navigational and Earth coordinates represented in figure 4 can be neglected. The position arises then as a result of simple integration of v^n . Figure 6 shows the trajectories angular and position error for one exemplary run. Also shown are the variances gathered by a Monte-Carlo-simulation with 500 runs. The simulated variance for the angular error amounts to less than 0.6° and the position error amounts to less than 0.04 m after 20 s. The combination of the random constant sensor fusion approach and the high

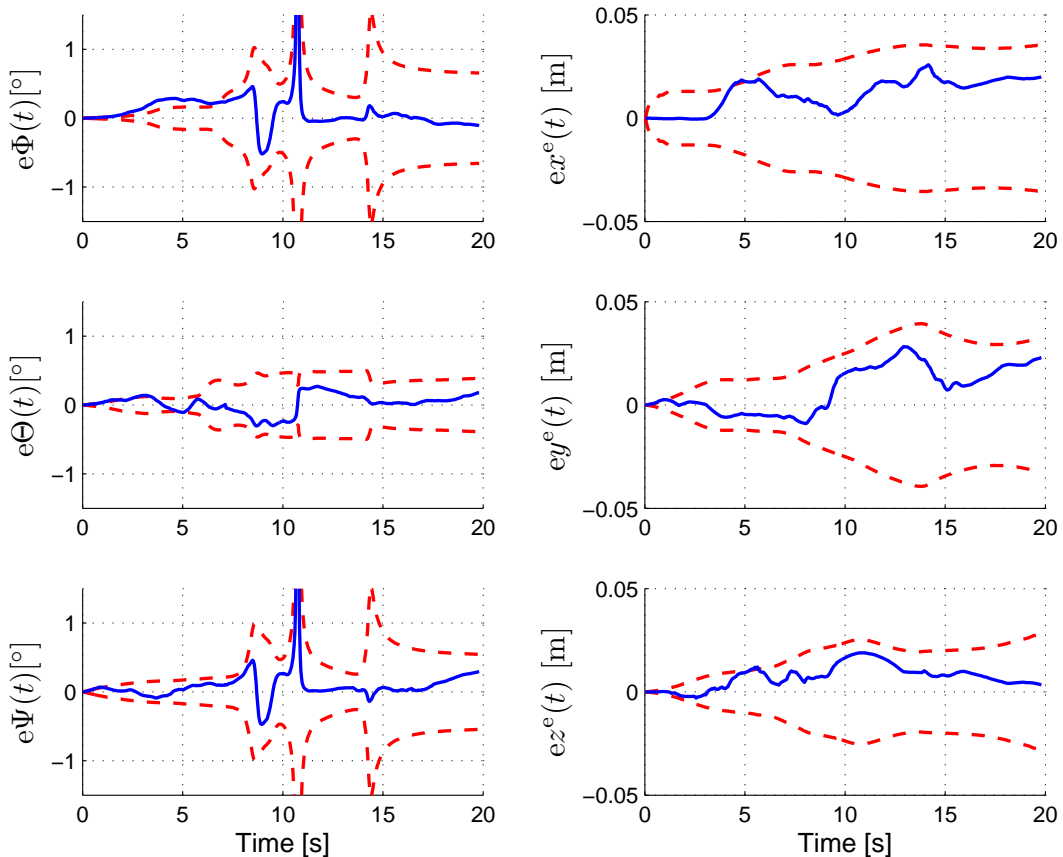


Figure 6. Angular (left) and Position (right) Errors for one sample run. The red dashed line denotes the optimal variance.

dynamic excitation of the rotational rates ($\dot{\Phi}^b(t), \dot{\Psi}^b(t)$) is leading to high variances at the timestamps 8, 11 and 14 seconds.

6. DISCUSSION

The proposed algorithm provides very good estimates of the navigational states when working on artificial flow fields with an variance of 10^{-2} pixel. State-of-the-art flow-field estimators fulfill these assumption.¹⁸ For a proper mode of work, IMOs have to be removed before passing our algorithm. Compared to the method of Raudies and Neumann⁶ the algorithm was enhanced for being able to estimate not only the magnitude but also the direction of the resulting velocity vector. Nevertheless the proposed framework cannot calculate the absolute value of the velocity due to its lack of measuring the corresponding depth. As mentioned earlier these can be overcome by using some distance measurement equipment such as stereoscopic vision or LASER-scanners. Having measured all 6 degrees of freedom with triple redundancy a Kalman filter is used for data fusion based on the on-line estimated covariances. The filter model is based on the assumption that the system does not change its states during one sample period. At the same time we assume a system variance of $N(0, \mathbf{Q})$. Work being undertaken addresses the model based prediction of navigational states which should lead to a better estimation performance compared to the random-constant approach described above. Moreover we would then be able to do system/sensor diagnosis. Compared to conventional Inertial Navigation Systems (INS) the used Strap-Down algorithm can be simplified quiet a lot. It reduces to a simple integrator chain with only one transformation between navigational and Earth coordinates if one prefers geodetic coordinates instead of a position in a Cartesian coordinate system. A big

advantage over an INS is that velocities have to be integrated only once to give a position not twice as it would be for accelerations.

7. CONCLUSION

In this paper we have shown that purely optical navigation is possible under certain circumstances. Such an *Optical Navigation System* (ONS) is based only on self measured information which makes the use of external sensors disposable for autonomous operations. Furthermore special inertial effects which have to be taken into account in INS does not have to be considered. Another advantage over an INS is the simplified strap-down algorithm where only velocities have to be integrated to result in position and attitude. We have shown that the ONS can compete with standard low-cost MEMS based inertial navigation systems. A minor drawback is the lack of doing a self alignment in unknown environments.

Having more than one camera on board all camera based sense-and-avoid or SLAM technologies can also be used without further augmenting the system design.

REFERENCES

- [1] Wendel, J., [*Integrierte Navigationssysteme*], Oldenburg-Verlag, München (2007).
- [2] J. Beyer, B. W., [*Grundlagen der Navigation*], Technische Universität Darmstadt, Darmstadt (2001).
- [3] Gibson, J., [*The Perception of Visual World*], Houghton Mifflin, Boston (1950).
- [4] A. R. Bruss, B. K. P. H., "Passive navigation," *Computer Vision, Graphics, and Image Processing* **21**, 3–20 (1983).
- [5] D. J. Heeger, A. D. J., "Subspace methods for recovering rigid motion i,"
- [6] F. Raudies, H. N., "An efficient linear method for estimation of ego-motion from optical flow,"
- [7] A. Giachetti, M. Campani, V. T., "The use of optical flow for road navigation,"
- [8] G. S. Klein, T. W. D., "A single-frame visual gyroscope," *16th British Machine Vision Conference (BMVC)*, 5–8 (2005).
- [9] A. Wu, E. Johnson, A. P., "Vision-aided inertial navigation for flight control," *AIAA Guidance, Navigation, and Control Conference and Exhibit* (2005).
- [10] O. Wulf, B. W., [*Fast 3D Scanning Methods for Laser Measurement Systems*], University of Hannover, Hannover (2003).
- [11] Scharstein, D. and Szeliski, R., "A taxonomy and evaluation of dense two-frame stereo correspondence algorithms," *International Journal of Computer Vision* **47**, 7–42 (2001).
- [12] A. Saxena, J. Schulte, A. Y. N., "Depth estimation using monocular and stereo cues," *Proc. IJCAI*, 2197–2203 (2007).
- [13] Weimer, F., [*Entwurf eines optischen Sensors zur Flugnavigation*], Institut für Flugmechanik und Flugregelung, Universität Stuttgart, Stuttgart (2007).
- [14] H. C. Longuet-Higgins, K. P., "The interpretation of a moving retinal image," *Proc. R. Soc. A* **208(1173)**, 385–397 (1980).
- [15] K Pauwels, M. M. V. H., "Robust instantaneous rigid motion estimation," *IEEE Computer Society Conference on Computer Vision and Pattern Recognition* **2**.
- [16] T. Zhang, C. T., "Fast, robust, and consistent camera motion estimation," *IEEE Computer Society Conference on Computer Vision and Pattern Recognition* **1**.
- [17] Gelb, A., [*Applied Optimal Estimation*], The MIT Press, Cambridge, Massachusetts (1974).
- [18] Bruhn, A., [*Variationelle Optische Flussberechnung - Präzise Modellierung und effiziente Numerik*], Fakultät für Mathematik und Informatik, Universität des Saarlandes, Saarland (2006).