DisCoverage: Distributed Optimization for Multi-Robot Exploration

TECHNISCHE UNIVERSITÄT DARMSTADT

A. Dominik Haumann, Kim D. Listmann, Jürgen Adamy
{dominik.haumann, kim.listmann}@rtr.tu-darmstadt.de

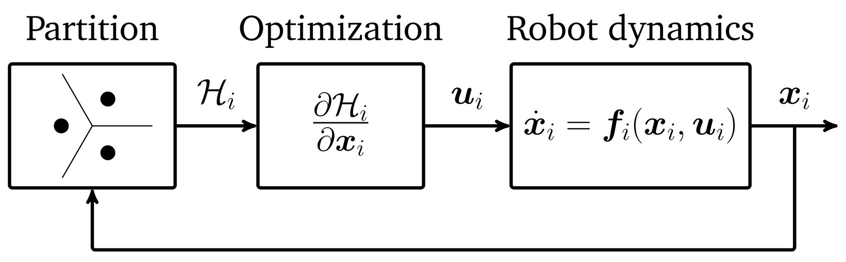
REGELUNGSTHEORIE UND ROBOTIK rtr

Motivation

Frontier-based [4] multi-robot exploration faces the challenge of assigning appropriate target points for each robot such that all robots explore different parts of the environment [3]. Existing strategies first determine those target points and *then* plan paths to reach the destinations, which reflects two consecutive steps. This article introduces *DisCoverage*: A new, distributed exploration strategy that merges both steps into a *single* one.

DisCoverage: Key Features

- distributed Voronoi partitioning of the environment
- distributed optimization of an objective function $\mathcal{H}_i$
- motion control laws $\boldsymbol{u}_i$ solving the exploration task



As this approach is an extension to the *coverage* problem [2] the proposed exploration strategy is referred as *DisCoverage*.

Simplifying Assumptions

- ideal omni-directional measurements
- reliable communication links
- all-to-all communication at any time
- convex, obstacle-free environment $\mathcal{Q} \subset \mathbb{R}^n$

Set-up and Definitions

- robot position $\boldsymbol{p}_i = \boldsymbol{x}_i \in \mathbb{R}^n$ with orientation $\delta_i \in [-\pi, \pi]$
- simple robot dynamics $\dot{\boldsymbol{p}}_i = \boldsymbol{u}_i$ with control input $\boldsymbol{u}_i$
- $\mathcal{P} := \{\boldsymbol{p}_1, \dots, \boldsymbol{p}_N\}$ and $\Delta := \{\delta_1, \delta_2, \dots, \delta_N\}$
- Voronoi cell $\mathcal{V}_i := \{\boldsymbol{q} \in \mathcal{Q} \mid \|\boldsymbol{q} - \boldsymbol{p}_i\|_2 \le \|\boldsymbol{q} - \boldsymbol{p}_j\|_2, \forall j\}$
- explored space $\mathcal{S} \subseteq \mathcal{Q}$ and $\mathcal{S}_i := \mathcal{S} \cap \mathcal{V}_i$, with
- $\partial\mathcal{S}_i$ denoting the frontiers in Voronoi cell $\mathcal{V}_i$

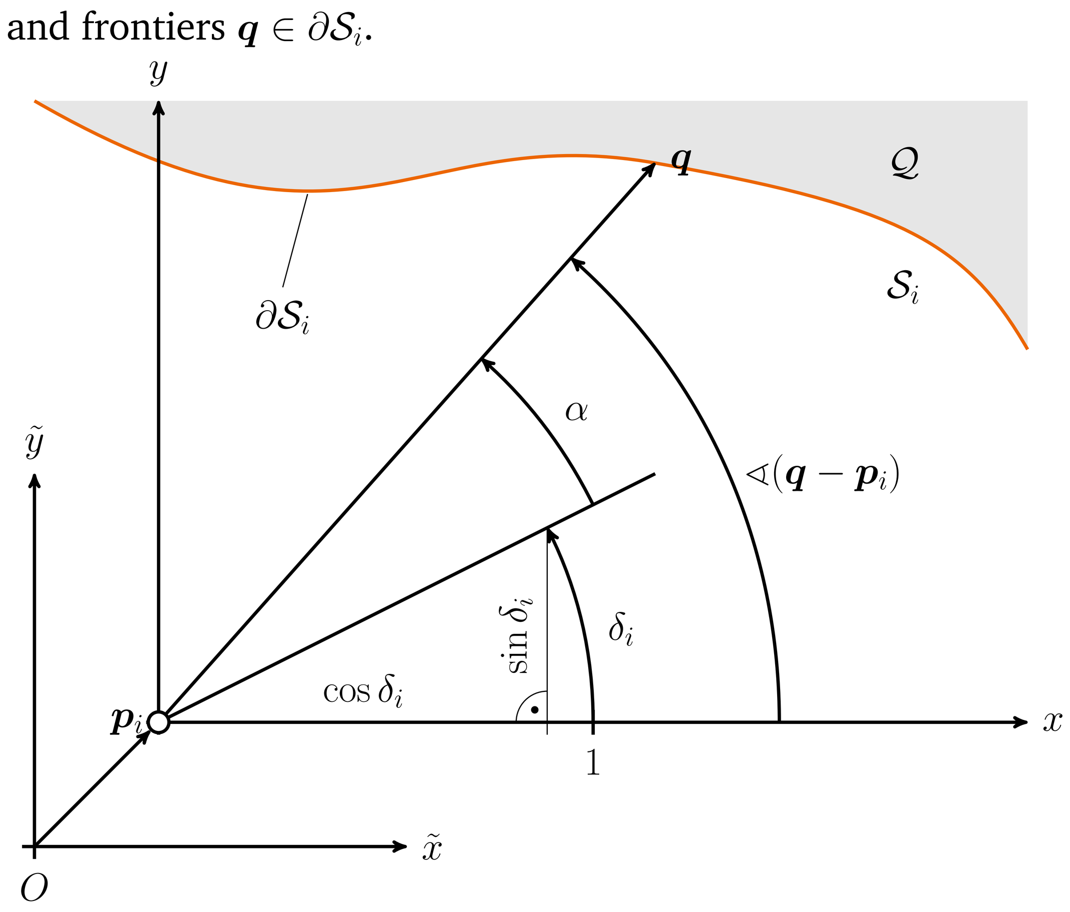
Objective Function

$$\mathcal{H}_{\text{discover}}(\mathcal{P}, \Delta) = \sum_{i=1}^{N} \mathcal{H}_{\text{discover},i}(\boldsymbol{p}_i, \delta_i) = \sum_{i=1}^{N} \int_{\partial\mathcal{S}_i} f(\boldsymbol{p}_i, \delta_i, \boldsymbol{q}) \phi(\boldsymbol{q}) d\boldsymbol{q}$$

- density function $\phi(\boldsymbol{q})$ encoding information gain in $\boldsymbol{q}$
- performance function $f(\cdot)$ given as

$$f(\boldsymbol{p}_i, \delta_i, \boldsymbol{q}) = \underbrace{\exp\left(-\frac{\alpha(\boldsymbol{p}_i, \delta_i, \boldsymbol{q})^2}{2\theta^2}\right)}_{\text{angular component}} \underbrace{\exp\left(-\frac{\|\boldsymbol{q} - \boldsymbol{p}_i\|_2^2}{2\sigma^2}\right)}_{\text{distance component}}$$

Goal: Find orientations δ_i such that as many frontiers $\boldsymbol{q} \in \partial\mathcal{S}_i$ as possible are located directly in front of each robot, i.e., minimize the angle $\alpha = \sphericalangle(\boldsymbol{q} - \boldsymbol{p}_i) - \delta_i$ between orientations δ_i and frontiers $\boldsymbol{q} \in \partial\mathcal{S}_i$.



Optimization Problem

- find optimal orientations $\delta_i \iff \mathcal{H}_{\text{discover}}(\mathcal{P}, \Delta) \to \max!$
- equivalent to distributed optimization

$$\delta_i^* = \arg\max_{\delta_i} \mathcal{H}_{\text{discover},i}(\boldsymbol{p}_i, \delta_i)$$

- necessary condition for each robot

$$\frac{\partial \mathcal{H}_{\text{discover},i}(\boldsymbol{p}_i, \delta_i)}{\partial \delta_i} = \int_{\partial\mathcal{S}_i} \frac{\partial f}{\partial \alpha} \frac{\partial \alpha}{\partial \delta_i} \phi(\boldsymbol{q}) d\boldsymbol{q} \overset{!}{=} 0$$

Motion Control Laws

- optimization yields optimal orientations δ_i^*
- simple first order dynamic system

$$\dot{\boldsymbol{p}}_i = \boldsymbol{u}_i = v \begin{pmatrix} \cos\delta_i^* \\ \sin\delta_i^* \end{pmatrix}$$

- constant velocity $v \in \mathbb{R}^+$
- closed loop, see first block diagram

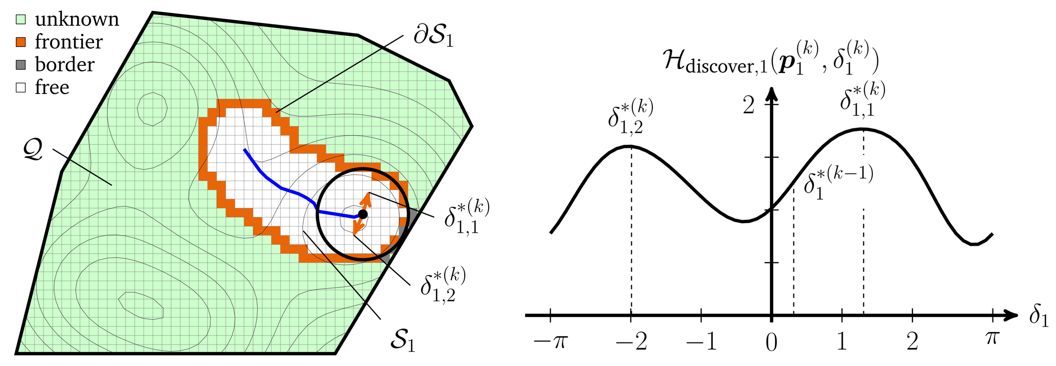
Implementation Details

- discrete time implementation in Matlab
- occupancy grid map, but arbitrary robot positions
- iteration index k

Algorithm:

Initialization of $\boldsymbol{p}_i^{(0)}$, $\delta_i^{(0)}$ and $\mathcal{S}^{(0)}$
while $\mathcal{S}^{(k)} \neq \mathcal{Q}$ **do**
  Communicate robot positions $\boldsymbol{p}_i^{(k)}$ and map data $\mathcal{S}^{(k)}$
  for $i = 1, \dots, N$ in parallel **do**
    Partitioning: $\mathcal{V}_i^{(k)}$ and $\partial\mathcal{S}_i^{(k)}$
    Optimization: $\delta_i^{*(k-1)} \longmapsto \delta_i^{*(k)}$
    Motion control: $\boldsymbol{p}_i^{(k)} \overset{\boldsymbol{u}_i}{\longmapsto} \boldsymbol{p}_i^{(k+1)}$
    Exploration: $\mathcal{S}_i^{(k)} \longmapsto \mathcal{S}_i^{(k+1)}$
  end for
end while

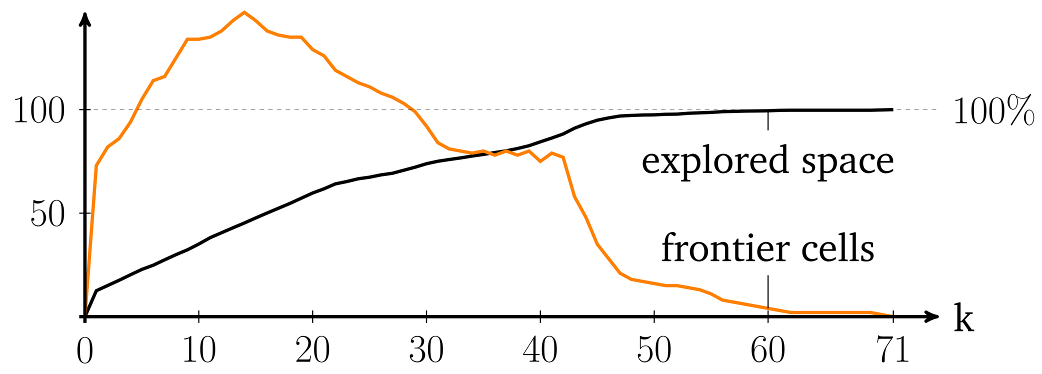
Single-Robot Exploration



⇒ Here, optimization yields two optimal orientations.

Multi-Robot Exploration

- coordinated exploration of the environment $\mathcal{Q}$ with 5 robots
- distributed optimization
- plot of exploration progress and amount of frontier cells

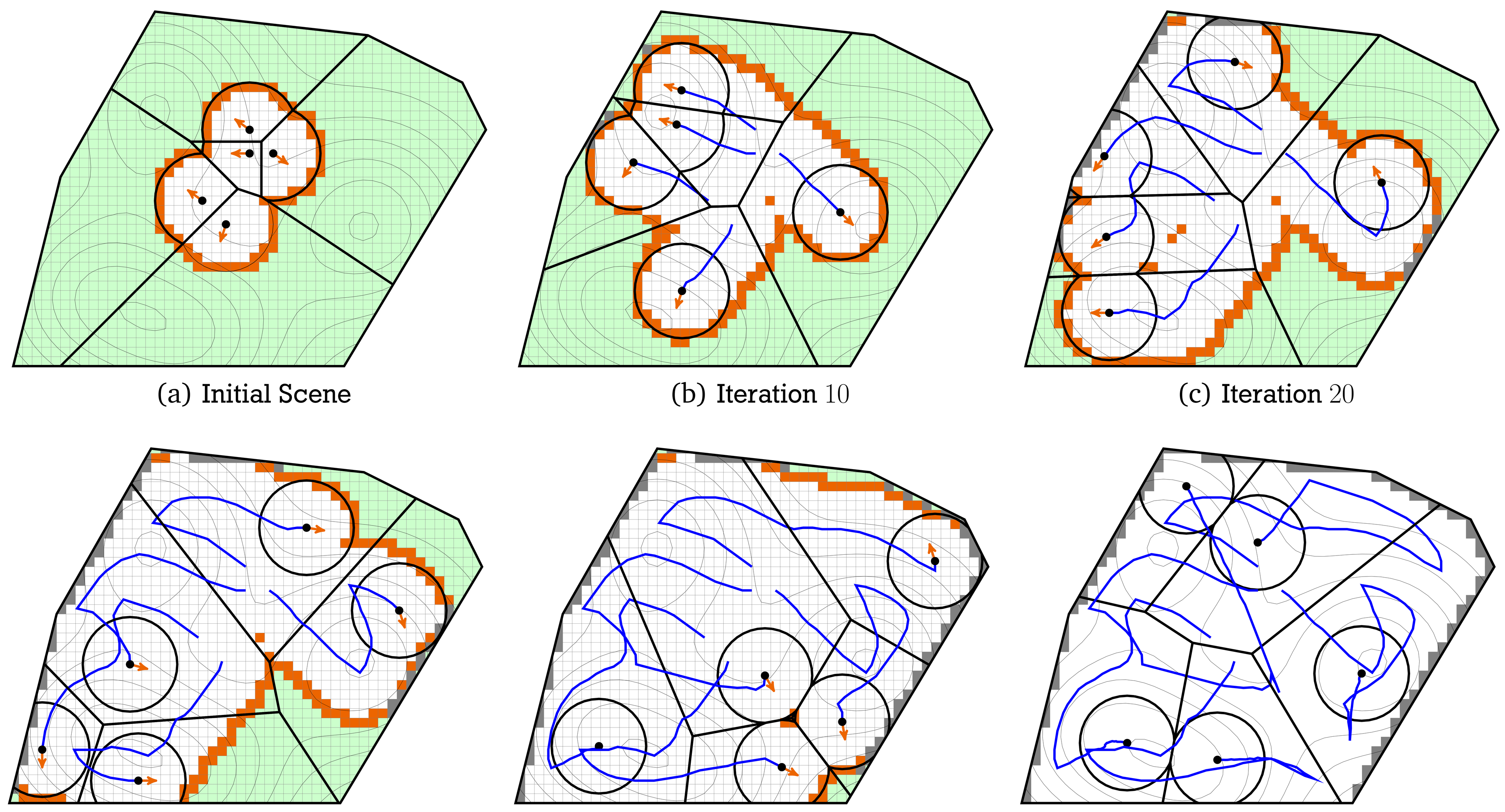


- exploration task complete after 71 iterations
- detailed visualization of the progress below

Conclusion

DisCoverage is a novel approach to multi-robot exploration merging the choice of target points and path planning into a single step. Based on a Voronoi partition, each robot optimizes a locally computable objective function to automatically obtain control vectors which assure simultaneous exploration of different regions of uncharted territory. The objective function is built such that it contains the orientation as degree of freedom in the optimization.

Detailed Results of the Multi-Robot Exploration Process



(a) Initial Scene (b) Iteration 10 (c) Iteration 20

(d) Iteration 30 (e) Iteration 45 (f) Iteration 71

References

[1] A. Gusrialdi et al. Coverage control for mobile networks with limited-range anisotropic sensors. In *Proc. IEEE CDC*, 2008.
[2] J. Cortés et al. Coverage control for mobile sensing networks. *IEEE Trans. on Robotics and Automation*, 20(2), 2004.
[3] W. Burgard et al. Coordinated multi-robot exploration. *IEEE Trans. on Robotics*, 21(3), 2005.
[4] B. Yamauchi. A frontier-based approach for autonomous exploration. In *Proc. IEEE CIRA*, 1997.

Gratefully supported by the German Research Foundation (DFG) within the GRK 1362 “Cooperative, Adaptive and Responsive Monitoring of Mixed Mode Environments” (`http://www.gkmm.tu-darmstadt.de`)