Purely Optical Navigation with Model Based State Prediction

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ABSTRACT

State-of-the-art Inertial Navigation Systems (INS) based on Micro-Electro-Mechanical Systems (MEMS) have a lack of precision especially in GPS denied environments like urban canyons or in pure indoor missions. The proposed Optical Navigation System (ONS) provides bias free ego-motion estimates using triple redundant sensor information. In combination with a model based state prediction our system is able to estimate velocity, position and attitude of an arbitrary aircraft. Simulating a high performance flow-field estimator the algorithm can compete with conventional low-cost INS. By using measured velocities instead of accelerations the system states drift behavior is not as distinctive as for an INS.

Keywords: Optical Navigation, Low-Cost, Model Based State Prediction, Optimal filtering, Data Fusion

1. INTRODUCTION

Recent investigation (Bryson and Sukkarieh,[1] Koifman and Bar-Itzhack,[2] Ma et al.[3]) have shown quite a big improvement of accuracy when aiding inertial navigation systems (INS) with a model of the vehicles dynamic behavior. The data-fusion of both the system dynamics and the autonomous INS information results in a more consistent and fault-tolerant solution of all navigational states. As pointed out by Bryson and Sukkarieh the main advantage of fusing model-knowledge and inertial measurements is the enhancement of observability of INS-errors as well as an improved fault tolerance. Using only information from autonomous sensors and system dynamics one can overcome the problems arising from GPS outtakes when a conventional INS-GPS integration technique is used. When having accurate sensors with a very small drift behavior such completely autonomous systems can achieve a quiet impressive accuracy and reliability. However problems will arise when using low-cost accelerometers and gyroscopes inside the INS.

As described by Graber et al.[4] it is possible to avoid such a distinctive drift behavior when using optical sensors instead of inertial sensors. This is mainly due to the fact of avoiding a double integration of accelerations to get positions since cameras measure velocities. Therefore a combination of such an optical sensor system and a model of the systems dynamics could arise in a more accurate navigation solution even with low-cost cameras.

Section 2 concerns on the modeling approach of the optical measurement system as well as the description of the flight dynamics and a short description of the simulation environment. The data fusion filter concept is explained in Sec. 3. To make the work complete some simulation results in Sec. 4 and a mandatory conclusion in Sec. 5 are given.

2. SYSTEM DESCRIPTION

Our proposed algorithm and system setup is shown schematically in Fig. 1. As we do not know more about the reality we model it through a pilot who is controlling an arbitrary vehicle through some input vector $\boldsymbol{u}$. The vehicles velocity and rotional rates can be measured by an optical 6 degree of freedom (6 DoF) estimation algorithm which is described in Sec. 2.1. On the other hand we have implemented models of the actuators and the flight mechanics which can be found in Sec. 2.2. The pilots model as explained in more detail in Sec. 3 simply adds some uncertainty to the inputs of the actuators due to not knowing the exact trajectory. Basically it is a

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random constant estimation algorithm updated through measurements of the systems input variables. The last block in Fig. 1 $\mathbf{C}_{\text{EOM}}$ selects the right system states from the state vector $\boldsymbol{x}_{\text{EOM}}$. As the proposed algorithm consists of a Kalmanfilter a feedback loop with a gain of $\mathbf{K}$ is included to correct the models internal states.

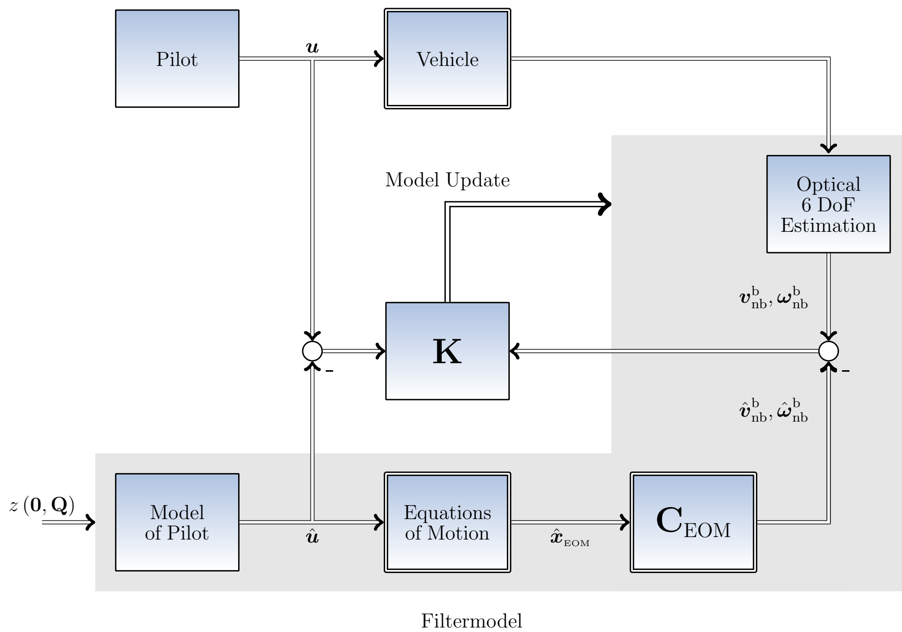

Figure 1. Proposed concept of sensor data fusion by the combination of a flight dynamics model with actuators, an optical measurement system as well as an pilot model for prediction of the models input values.

2.1 Optical 6 DoF Estimation

Under the assumption of a rigid environment the observers (respectively vehicles) ego motion can be expressed by pixel displacement vectors. The displacement vectors are also labeled as optical flow[5] $\boldsymbol{u} = [U_{\text{x}}\ U_{\text{y}}]^{\text{T}}$. Following this approach Louget-Higgens and Prazdney[6] derived an instantaneous ego motion model which relates the optical flow $\mathbf{U}$ to the observers translational velocity $\boldsymbol{v}^{\text{c}}_{\text{nc}}$ and rotational rate $\boldsymbol{\omega}^{\text{c}}_{\text{nc}}$.

This model uses a pinhole camera with the focal length f and the transform matrix $\mathbf{C}^{\text{c}}_{\text{n}}$ which projects 3-dimensional points $\boldsymbol{p}^{\text{n}} = [x^{\text{n}}\ y^{\text{n}}\ z^{\text{n}}]^{\text{T}}$ onto a 2-dimensional image plane μ

$$\boldsymbol{p}^{\text{c}} = \mathbf{C}^{\text{c}}_{\text{n}} \cdot\ \boldsymbol{p}^{\text{n}} \tag{1a}$$

$$\boldsymbol{p}^{\text{p}} = \begin{bmatrix} x^{\text{p}} \\ y^{\text{p}} \end{bmatrix} = \frac{z^{\text{c}}}{f} \cdot \begin{bmatrix} x^{\text{c}} \\ y^{\text{c}} \end{bmatrix} \tag{1b}$$

like it is shown in figure 2. The observers motion causes a instantaneous displacement $\dot{\boldsymbol{p}}^{\text{c}} = -\boldsymbol{v}^{\text{c}}_{\text{nc}} - \boldsymbol{\omega}^{\text{c}}_{\text{nc}} \times \boldsymbol{p}^{\text{c}}$ of each pixel. Using this model, movements of projected points on the 2-dimensional image plane have the velocity

$$\mathbf{U} = \begin{bmatrix} U_{\text{x}} \\ U_{\text{y}} \end{bmatrix} = \frac{1}{z^{\text{c}}} \begin{bmatrix} -f & 0 & x^{\text{p}} \\ 0 & -f & y^{\text{p}} \end{bmatrix} \boldsymbol{v}^{\text{c}}_{\text{nc}} + \frac{1}{f} \begin{bmatrix} x^{\text{p}} y^{\text{p}} & -\left(f^2 + (x^{\text{p}})^2\right) & f y^{\text{p}} \\ \left(f^2 + (x^{\text{p}})^2\right) & -x^{\text{p}} y^{\text{p}} & -f x^{\text{p}} \end{bmatrix} \boldsymbol{\omega}^{\text{c}}_{\text{nc}} \tag{2}$$

An important observation about Eq. 2 is that it is bilinear. The optical flow $\mathbf{U}$ is a linear function of $\boldsymbol{v}^{\text{c}}_{\text{nc}}$ and $\boldsymbol{\omega}^{\text{c}}_{\text{nc}}$ for a fixed depth z, and it is a linear function of z^{c} and $\boldsymbol{\omega}^{\text{c}}_{\text{nc}}$ for fixed $\boldsymbol{v}^{\text{c}}_{\text{nc}}$. Since both z^{c} and $\boldsymbol{v}^{\text{c}}_{\text{nc}}$ are unknowns and since they are multiplied by each other they can only be determined up to a scale factor $\kappa \in \mathbb{R}$.

$$\boldsymbol{v}^{\text{c}}_{\text{nc}} = \kappa \cdot \bar{\boldsymbol{v}}^{\text{c}}_{\text{nc}} \tag{3}$$

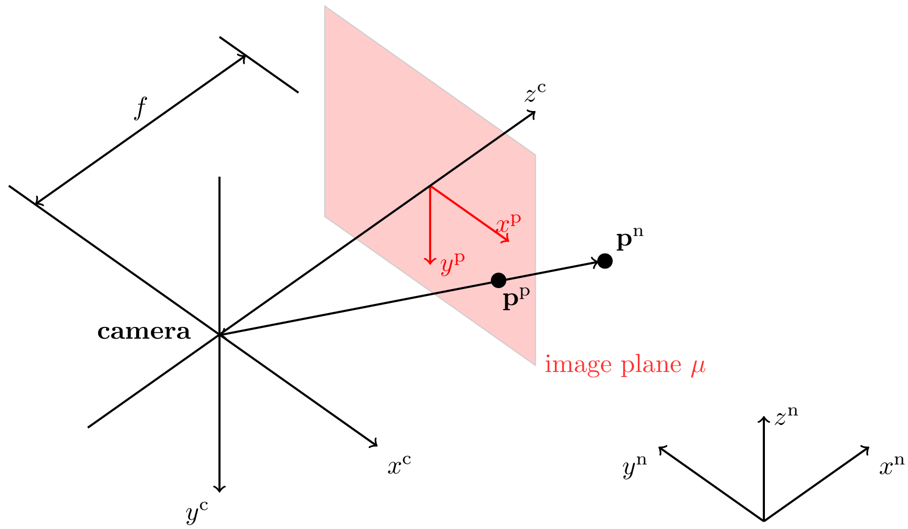

Figure 2. Projective geometry for a pinhole camera

Where $\bar{\boldsymbol{v}}$ denotes a unity vector

Using Eq. (2) Bruss and Horn[7] formulated an depth independent nonlinear constraint to get rid off the unknown depth z. This results in the well known *bilinear optimization constraint*, wich is formulated mathematically by Heeger and Jepson[8] through

$$\mathbf{0} = \boldsymbol{v}^{\mathrm{c}}_{\mathrm{nc}} \left(\mathbf{M} - \mathbf{H} \boldsymbol{\omega}^{\mathrm{c}}_{\mathrm{nc}} \right) \tag{4}$$

where

$$\mathbf{M} = \begin{bmatrix} fU_{\mathrm{x}} \\ -fU_{\mathrm{y}} \\ y^{\mathrm{p}} U_{\mathrm{x}} - x^{\mathrm{p}} U_{\mathrm{y}} \end{bmatrix} \tag{5a}$$

$$\mathbf{H} = \begin{bmatrix} -\left(f^2 + x^{\mathrm{p}2}\right) & x^{\mathrm{b}} y^{\mathrm{p}} & f x^{\mathrm{p}} \\ x^{\mathrm{p}} y^{\mathrm{p}} & -\left(f^2 + x^{\mathrm{p}2}\right) & f y^{\mathrm{p}} \\ f x^{\mathrm{p}} & f y^{\mathrm{p}} & -\left(x^{\mathrm{p}2} + y^{\mathrm{p}2}\right) \end{bmatrix} \tag{5b}$$

Based on Eq. (4) there are numerous publications which allow an estimate of 6 DoF ego motion: (i) Heeger and Jepson[8] used this equation to built their subspace method. (ii) Pauwels and Van Hulle[9] used an iteration mechanism optimizing for rotation and translation. (iii) Zhang and Tomasi[10] as well as Pauwels and Van Hulle[11] used a Gauss-Newton iteration between rotation, depth, and translation. (iv) Raudies and Neumann[12] transfered it into a linear optimization problem by introduction of auxiliary variables. All these approaches have in common that they permit a 6 DoF estimation with a monocular camera.

In our investigation we extended the linear approach of Raudies and Neumann to overcome a minor drawback of their approach. As descriped in Graber et al.[4] we added a further step to estimate the sign of the unit velocity vector $\bar{\boldsymbol{v}}^{\mathrm{c}}_{\mathrm{nc}}$.

In a next step we assume an available depth measurement for each optical flow vector so we can obtain an estimate of the absolute velocity $\boldsymbol{v}^{\mathrm{c}}_{\mathrm{nc}}$. Inserting Eq. (3) into Eq. (2) and using the existing ego motion estimates $(\hat{\boldsymbol{v}}^{\mathrm{c}}_{\mathrm{nc}}, \hat{\boldsymbol{\omega}}^{\mathrm{c}}_{\mathrm{nc}})$ with the addional depth measurement $y_{\mathrm{depth}} = z^{\mathrm{c}}$ results in a modeled optical flow $\hat{\mathbf{U}}$. Then a linear optimization problem by minimizing the Euclidean distance of the residual between the estimated flow vector $\mathbf{u} = \left[U_{\mathrm{x}}\ U_{\mathrm{y}}\right]^{\mathrm{T}}$ from a spatio-temporal image sequence and the modeled flow vector $\hat{\mathbf{U}}$ with respect to the scale factor κ can be formulated as

$$R(\kappa) = \left|\left| \mathbf{U} - \hat{\mathbf{U}}(\kappa) \right|\right|_2^2 \xrightarrow{\kappa} \min \qquad \text{with} \qquad \kappa \in \{0, \infty\} \tag{6}$$

With an estimate of κ and Eq. (3) we then obtain an estimate of the absolute velocity $\boldsymbol{v}^{\mathrm{c}}_{\mathrm{nc}}$. Finally, the resulting translational velocity and rotational rate in camera coordinates have to be transfered into navigational coordinates through

$$\begin{aligned}\boldsymbol{v}^{\mathrm{b}}_{\mathrm{nb}} &= \mathbf{C}^{\mathrm{b}}_{\mathrm{n}} \cdot \mathbf{C}^{\mathrm{n}}_{\mathrm{c}} \cdot \boldsymbol{v}^{\mathrm{c}}_{\mathrm{nc}} \\ \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}} &= \mathbf{C}^{\mathrm{b}}_{\mathrm{n}} \cdot \mathbf{C}^{\mathrm{n}}_{\mathrm{c}} \cdot \boldsymbol{\omega}^{\mathrm{c}}_{\mathrm{nc}}.\end{aligned}$$

Therein the matrix $\mathbf{C}^{\mathrm{b}}_{\mathrm{n}} = (\mathbf{C}^{\mathrm{n}}_{\mathrm{b}})^{\mathrm{T}}$ transforms variables from navigational into body coordinates.

2.2 Equations of Motion

For our investigation we used a simplified dynamic model of a quadrotor helicopter as described for example by Hoffmann et al.[13] Some of the therein mentioned aerodynamic effects were neglected due to a lighter model complexity for a first discussion. Basically the model is driven by four DC-motors with a constant voltage to thrust ratio K_{F} and voltage to torque ratio K_{M}. The input vector is given by $\boldsymbol{u} = [U_1\ U_2\ U_3\ U_4]^{\mathrm{T}}$.

The state vector is given by $\boldsymbol{x} = [\boldsymbol{p}^{\mathrm{n}}\ \boldsymbol{v}^{\mathrm{b}}_{\mathrm{nb}}\ \boldsymbol{\Phi}\ \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}}]^{\mathrm{T}}$ which are the position in a local Cartesian coordinate system, the vehicles body velocity, Euler angles and the rotational rates of the quadrotor. Aerodynamic drag forces and torques are only linear depended on the velocity. These assumptions lead to a system of 12 differential equations $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$ shown in Eq. (7).

$$\dot{\boldsymbol{p}}^{\mathrm{n}} = \mathbf{C}^{\mathrm{n}}_{\mathrm{b}} \cdot \boldsymbol{v}^{\mathrm{b}}_{\mathrm{nb}} \tag{7a}$$

$$\dot{\boldsymbol{v}}^{\mathrm{b}}_{\mathrm{nb}} = \frac{\boldsymbol{F}^{\mathrm{b}}}{m} + \boldsymbol{v}^{\mathrm{b}}_{\mathrm{nb}} \times \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}} \tag{7b}$$

$$\dot{\boldsymbol{\Phi}} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \cdot \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}} \tag{7c}$$

$$\dot{\boldsymbol{\omega}}^{\mathrm{b}}_{\mathrm{nb}} = \left[\boldsymbol{M}^{\mathrm{b}} - \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}} \times \mathbf{I}\boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}}\right] \cdot \mathbf{I}^{-1} \tag{7d}$$

where

$$\boldsymbol{F}^{\mathrm{b}} = -\boldsymbol{F}^{\mathrm{b}}_{\mathrm{p}}(\boldsymbol{u}) + \boldsymbol{F}^{\mathrm{b}}_{\mathrm{g}} - \boldsymbol{F}^{\mathrm{b}}_{\mathrm{a}} \tag{8a}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ U_1 + U_2 + U_3 + U_4 \end{bmatrix} \cdot K_{\mathrm{F}} + \mathbf{C}^{\mathrm{b}}_{\mathrm{n}} \cdot \begin{bmatrix} 0 \\ 0 \\ m \cdot g \end{bmatrix} - \begin{bmatrix} C_{\mathrm{F,xy}} & 0 & 0 \\ 0 & C_{\mathrm{F,xy}} & 0 \\ 0 & 0 & C_{\mathrm{F,z}} \end{bmatrix} \cdot \boldsymbol{v}^{\mathrm{b}}_{\mathrm{nb}} \tag{8b}$$

are the forces generated by the propulsion system, gravity and aerodynamic effects all given in body coordinates. The C_F terms describe the aerodynamic drag coefficients in the accordant direction. To make the dynamic equations complete the forcing torque is calculated by

$$\boldsymbol{M}^{\mathrm{b}} = \boldsymbol{M}^{\mathrm{b}}_{\mathrm{p}}(\boldsymbol{u}) - \boldsymbol{M}^{\mathrm{b}}_{\mathrm{a}} \tag{9a}$$

$$= \begin{bmatrix} (U_4 - U_2) \cdot K_{\mathrm{F}} l_{\mathrm{m}} \\ (U_1 - U_3) \cdot K_{\mathrm{F}} l_{\mathrm{m}} \\ (U_1 - U_2 + U_3 - U_4) \cdot K_{\mathrm{M}} \end{bmatrix} - \begin{bmatrix} C_{\mathrm{M,xy}} & 0 & 0 \\ 0 & C_{\mathrm{M,xy}} & 0 \\ 0 & 0 & C_{\mathrm{M,z}} \end{bmatrix} \cdot \boldsymbol{\omega}^{\mathrm{b}}_{\mathrm{nb}} \tag{9b}$$

with the distance l_{m} between each motor and the center of gravity and the respective aerodynamic torque coefficients C_M. Systems inertia is described by the purely diagonal matrix $\mathbf{I}$

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{10}$$

which contains the moments of inertia in body fixed x, y and z axis. Due to the quadrotors limited roll and pitch angles we safely can rely on Euler angles for calculating the transformation matrices $\mathbf{C}^{\mathrm{n}}_{\mathrm{b}}$ and $\mathbf{C}^{\mathrm{b}}_{\mathrm{n}}$ as well as the vehicles attitude.

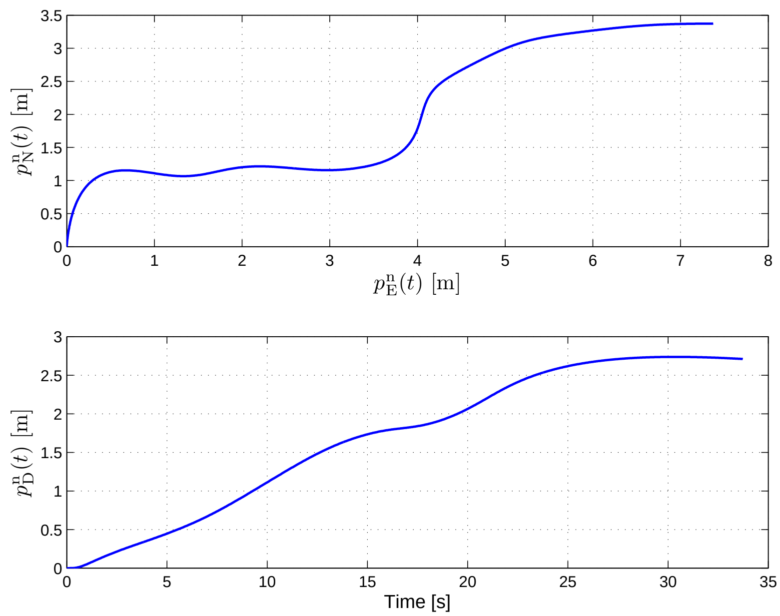

Figure 3. The used flight path for simulation. In the upper part of the figure the topview is shown while the lower part depicts the height profile of the trajectory.

2.3 Simulation description

For analyzing the performance of the proposed approach a simulation environment was developed. It consists of a trajectory generator for the quadrotor which simply guides the vehicle along a given flight path, see figure 3. To have all measurements for the Kalman filtering afterwards, the input voltages are logged as well as all system states of the reference trajectory.

The simulated system states are then fed to the optical measurement system, which consists of three monocular cameras pointing to each of the body axes. The simulated cameras feature a resulution of 512×512 pixel and a field of view of $60°$. Furthermore we are using a simulated sparse optical flow field based on Eq. (2) with 100 features and an additional white noise with mean $\mu = 0$ and deviation of $\sigma = 0.1$ pixel. This assumption is true for modern optical flow estimators like the approach of Bruhn.[14] The depth measurement is simulated as an additional white noise with a depth depending deviation of $\sigma = 0.3 \cdot z^{\mathrm{c}}$ meter and mean $\mu = 0$.

As discribed in Sec. 2.1 one camera is able to estimate the 6 DoF motion. While we are using three cameras we archive a triple redundant estimate for the vehicles velocity and angular rates. Besides there is a Kalmanfilter as described in Sec. 3 which does the data fusion of all measured values. Beforehand the measured voltages are perturbed by some noise to simulate non ideal sensors. To distinguish between reality and the quadrotor model, we have slightly changed parameters for the equations of motion. These changed parameters result in an eigenvalue deviation of $\pm 3\%$.

3. FILTER CONCEPT

In this section the chosen filter approach is described. While having an overall of ten measurements available to correct the internal 16 system states which depend partially nonlinear on each other we decided to use an Extended Kalman Filter (EKF) for data fusion. As the EKF equations[15] are well known we desist from explaining them here. But for convenience they are printed down in Eqs. (11) for the prediction step and in Eqs. (12) for the correction step.

$$\hat{\boldsymbol{x}}_k^- = f(\hat{\boldsymbol{x}}_{k-1}, \mathbf{0}) + \boldsymbol{w}_k \tag{11a}$$

$$\hat{\mathbf{P}}_k^- = \mathbf{A}_k \hat{\mathbf{P}}_{k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q} \tag{11b}$$

Afterwards the *a priori* state and covariance estimates are updated by the measurement $\boldsymbol{y} = \left[\boldsymbol{v}_{\text{nb}}^{\text{b}}\ \boldsymbol{\omega}_{\text{nb}}^{\text{b}}\ \boldsymbol{u}\right]^{\text{T}}$:

$$\mathbf{K}_k = \mathbf{P}_k^{-}\mathbf{C}^{\text{T}}\left(\mathbf{C}\hat{\mathbf{P}}_k^{-}\mathbf{C}^{\text{T}} + \mathbf{R}\right)^{-1} \tag{12a}$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^{-} + \mathbf{K}_k\left(\boldsymbol{y} - \mathbf{C}\hat{\boldsymbol{x}}_k^{-}\right) \tag{12b}$$

$$\hat{\mathbf{P}}_k = \left(\mathbf{I} - \mathbf{K}_k\mathbf{C}\right)\hat{\mathbf{P}}_k^{-} \tag{12c}$$

For performing a state prediction with an EKF one has to set up the complete system matrix $\mathbf{A} \in \mathbb{R}^{16\times 16}$ as shown in Eq. (13). The output matrix $\mathbf{C}$ selects the measured states from $\hat{\boldsymbol{x}}$ according to Eq. (14).

$$\mathbf{A}_k = \left.\frac{d\boldsymbol{f}}{d\boldsymbol{x}}\right|_{x_k^{-}} \tag{13}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}^{3\times 3} & \mathbf{1}^{3\times 3} & \mathbf{0}^{3\times 3} & \mathbf{0}^{3\times 3} & \mathbf{0}^{3\times 4} \\ \mathbf{0}^{3\times 3} & \mathbf{0}^{3\times 3} & \mathbf{0}^{3\times 3} & \mathbf{1}^{3\times 3} & \mathbf{0}^{3\times 4} \\ \mathbf{0}^{4\times 3} & \mathbf{0}^{4\times 3} & \mathbf{0}^{4\times 3} & \mathbf{0}^{4\times 3} & \mathbf{1}^{4\times 4} \end{bmatrix} \tag{14}$$

with $\mathbf{0}$ an all zero matrix and $\mathbf{1}$ a matrix with 1 on its diagonal entries. The respective dimensions are given for every single block matrix inside $\mathbf{C}$.

The first 12 elements of $\hat{\boldsymbol{x}}$ belong to the quadrotors equations of motion, i.e. $\hat{\boldsymbol{x}}_{\text{EOM}}$. The dynamic behavior is described in Eq.(7). The last 4 entries of the state vector describe the input voltages of the 4 motors. The dynamic behavior is set to zero as we presume no change during two time steps. This is called a Random Constant (RC) estimation approach. During the correction step of the Kalmanfilter these voltages are updated by the real measured motor voltages. Knowing not more about the pilots behavior this seems to be the best guess for a first investigation. Work being undertaken concerns on a more appropriate modeling of the pilots behavior to result in a better accuracy of the systems input signals $\hat{\boldsymbol{u}}$.

The uncertainty in modeling this way is taken into account by a suitable parameterization of the systems noise matrix $\mathbf{Q}$ as usual in Kalmanfilter theory. For a parameterization of the measurement covariance matrix $\mathbf{R}$ the voltage sensors and cameras characteristic behavior is analyzed. We have assumed 10bit analog-digital voltage converters where the least significant bit (LSB) is not stable. Therefore the variance of the measured voltage is given by $\sigma_{\boldsymbol{u}}^2 = (U_{\text{max}}/(2^{10}-1))^2 \approx 2\cdot 10^{-4}$ with $U_{\text{max}} = 14.8\text{V}$. The other values of $\mathbf{R}$ are given by the optical measurment system for our chosen accuracy. The initial systems covariance matrix $\mathbf{P}_0$ is chosen appropriate. This leads to an assignment of $\mathbf{P}_0$, $\mathbf{Q}$ and $\mathbf{R}$ as follows:

$$\mathbf{P}_0 = 0.1\cdot\mathbf{I}^{16\times 16}$$

$$\mathbf{Q} = diag\left(10^{-3}, 10^{-3}, 10^{-3}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-5}, 10^{-4}, 10^{-2}, 10^{-3}, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}\right)$$

$$\mathbf{R} = diag\left(10^{-4}, 10^{-4}, 10^{-4}, 10^{-2}, 10^{-2}, 10^{-2}, 2\cdot 10^{-4}, 2\cdot 10^{-4}, 2\cdot 10^{-4}, 2\cdot 10^{-4}\right)$$

To cope with the time shift between an applied voltage and the therefore changing system states we use the values for $\hat{\boldsymbol{u}}$ of the preceding time step, i.e. $\hat{\boldsymbol{x}}_{k-1,13:16}$, to predict all other states.

Another important fact to be mentioned is that before correcting the state estimates a sensor data fusion of the three redundant measured velocities and rotational rates is done with an optimal signal blending. Let σ be the deviation between a measured variable z and an *a priori* estimate $\hat{z}$ of the corresponding variable. Then the optimal blended estimate $\hat{z}_{\text{opt}}$ is given by Eq. (15).

$$\sigma_i^2 = (\hat{z} - z_i)^2$$

$$\hat{z}_{\text{opt}} = \frac{\sigma_2^2\sigma_3^2 z_1 + \sigma_1^2\sigma_3^2 z_2 + \sigma_1^2\sigma_2^2 z_3}{\sigma_1^2\sigma_2^2 + \sigma_1^2\sigma_3^2 + \sigma_2^2\sigma_3^2} \tag{15}$$

All of these optimal blended informations are then fed as measurement $\boldsymbol{y}$ into the correction step of the EKF.

The chosen approach completely separates the "real world" from the system description as originally postulated by Kalman[16] in 1960. Shortly said Kalman claims that a systems output shall be predictable by a dynamic model which could be driven by White Gaussian noise. A recently finished diploma thesis by Stefan Heindel[17] at our institute has shown the potential when implementing a Kalmanfilter the way it was supposed to. In a preceding literature inquiry he has not found anything comparable where Kalmans system requirements were completely fulfilled.

4. RESULTS

In this section some simulation results are described. Due to the model based state estimation it was possible to improve the accuracy compared to a estimation technique which only uses measurements of the system states, i.e. velocities and rotational rates.[4] This was our firstly chosen implementation. The herein used approach which also considers the input voltages of the motors outperforms the former one. Especially the roll and pitch angles (ϕ and θ) can be estimated much better without drifting away as time advances. This is founded in the Kalmanfilters theory where a correlation between measured velocities in x or y-direction and roll or pitch angle is build up during the filtering process. As one has a quadrotor in mind it becomes clear that velocities in x or y-direction can only be built up when the vehicle is tilted by some angle. The filters behaviour therefore coincides with the flight mechanics of the vehicle.

For a graphical presentation of the errors have a look at Fig. 4. As one can see the errors boundaries (dashed lines) for the positions are growing bigger over time. This is mainly due to integrating the measured velocities for these respectively variables which always results in a random walk behavior. The heading angle cannot be estimated as good as the attitude angles. On the one hand there is no absolute reference value for it and on the other hand this angle cannot be aided by any measured velocity like it is true for ϕ or θ. Here the system has to depend on the integrated rotational rate around the body-fixed z-axis.

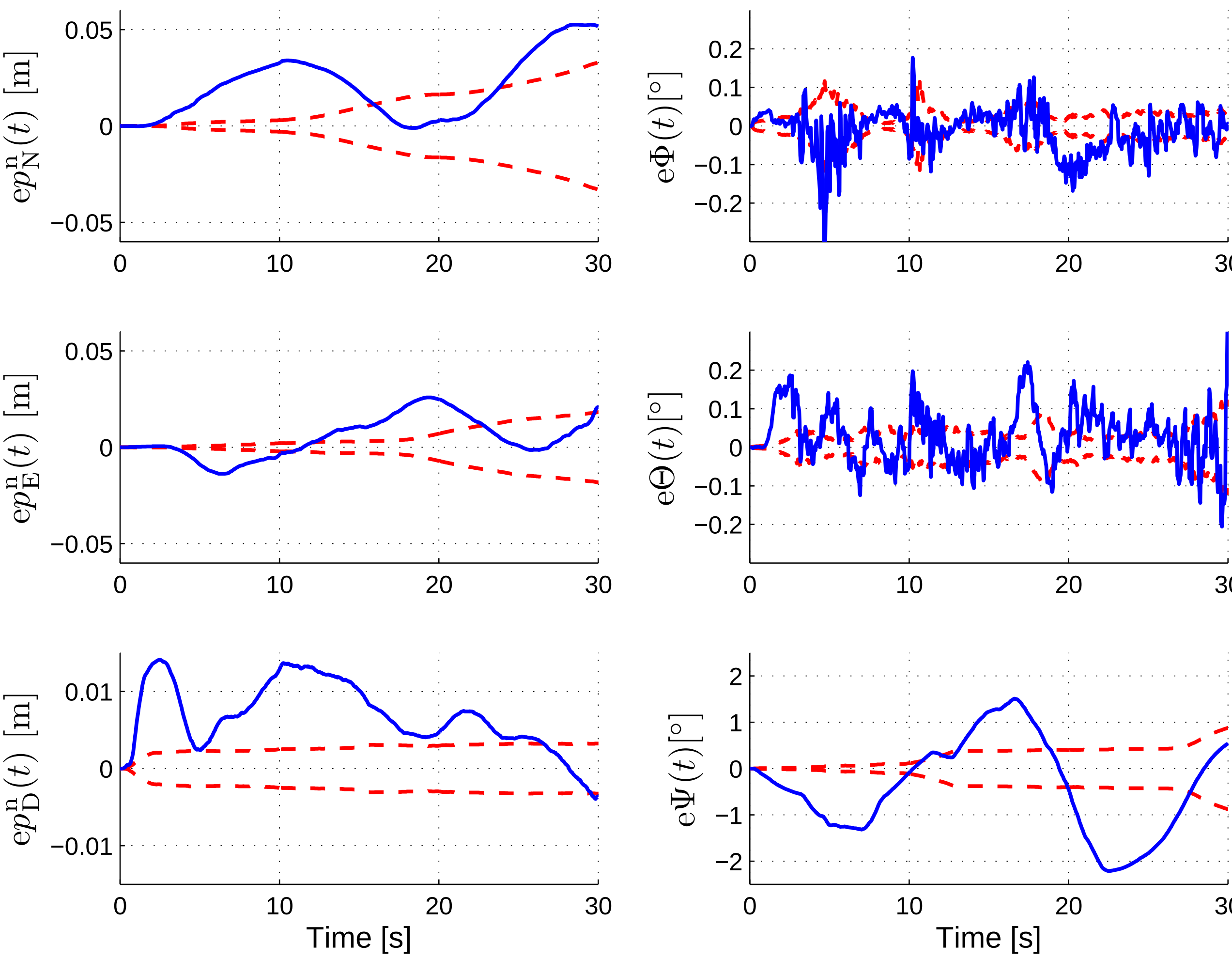

Figure 4. Errors between estimated and true navigation states. The dashed lines describe the $\pm 1\sigma$ error boundaries of the estimates states. They were estimated by a Monte-Carlo simulation over 50 runs with changing seeds of the random number generators. An exemplary simulation run is shown by the solid line.

5. CONCLUSION

Low-cost and light weight navigation equipment is subject of research in the field of autonomous operating vehicles. Up to now the research mainly focuses on inertial measurement units as they are small and comparable low priced. Sometimes an aiding by optical systems is used to cope with the tremendous drift behavior of inertial sensors. In this work we have described a completely autonomous navigation system using only optical information for estimating the well known navigational states, position, attitude and velocity of an arbitrary vehicle. Based on a simplified model of a quadrotor helicopter the performance of such a system was shown by simulations. Our system is able to estimate velocity and rotational rate of an vehicle without drift by a system of three cameras. Position and attitude calculated from these data do not have the distinctive random walk characteristics of inertial sensor based systems. Especially ϕ and θ do not have any random walk behavior. Also position and attitude information depend on each other inside the optical measurement system which makes the whole algorithm more consistent and fault tolerant to measurement errors. By the use of a pilot model it is possible to predict the models output more realistic as it would be when using only measurements of the system velocity and rotation rate.

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