MILP-basierte Optimalsteuerung kooperativer Multi-Vehikel-Systeme

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Examples of cooperative multi-robots / multi-vehicle systems

- Cooperative fire surveillance
  - Source: www.aware-project.net

- Mobile assistance in wireless sensor networks

- Cooperative robot games
  - Source: www.robocup.org

- Warehouse automation
  - www.kivasytems.com
Cooperating mobile robots / vehicles: discrete decisions and trajectory planning

Robot team competition:
Tight coupling of
- robots’ motion dynamics and
- role distribution,
- task assignment.

Monitoring with unmanned aerial vehicles:
Tight coupling of
- vehicle specific dynamics and
- target assignment,
- waypoint sequencing.

source: www.robocup.org

source: www.comets-uavs.org
Current research topics

- appropriate modeling of the multi-vehicle system
- model reduction and abstraction
- system analysis and optimization
- planning and control methods for cooperative multi-vehicle-systems
- application to heterogeneous hardware and various scenarios

⇒ optimal control of cooperation
⇒ consistent bridging of the gap between "exact" methods and heuristic approaches
Modeling Cooperative Multi-Vehicle Systems

MILP-based Optimal Control of Cooperating Multi-Vehicle Systems

Towards On-line MPC

Summary and Outlook
Problem formulation:

some characterizing details

single vehicle $i$:

- locomotion
  \[ \dot{x}^i = f^i_{q_i}(x^i(t), u^i(t)) \]

- constraints
  \[ g^{i}_{q_i}(x^i(t), u^i(t)) \leq 0 \]

- allowable sequences of tasks/roles:
  \[ r^i(b^i_{q_i}(t_s - 0), b^i_{q_i}(t_s + 0)) \leq 0 \]
  $b^i_{q_i}(t) \in \{0, 1\}$, $q_i \in \{1, ..., L_i\}$
  $t_s \in \{t_k|k = 1, ..., n_s\}$: switching time

...and a diagram of a vehicle moving in the x-direction.

cooperative system $(i_1, i_2 \in \{1, ..., n_v\})$:

- collision avoidance
  \[ g_{coll}(x^{i_1}(t), x^{i_2}(t)) \leq 0 \]

- underlying geometrical structure
  binary variables: $b_q(t) \in \{0, 1\}$
  mixed constraints
  \[ [b_q(t) = 1 \Rightarrow C_q(x^{i_1}(t), x^{i_2}(t)) \leq 0] \]

- optimal cooperation
  \[ \Phi(x(t), u(t), b(t)) \rightarrow \min \]
Hybrid automata

\[ H = (V, E, X, U, ini, f, j, i, e) \]

- **Basic components** [Henzinger, 1996]:
  - \((V, E)\): finite directed multigraph with knots in \(V\) (states) and edges in \(E\) (switches)
  - \(X\): set of continuous state variables
  - \(U\): set of continuous control variables
  - \(ini\): map which assigns an initial condition to each edge
  - \(i\): assigns feasible region of state variables to each knot (inequality, equality constraints)
  - \(f\): flow equation or state dynamics for each state
  - \(j\): jump conditions at edges
  - \(e\): events at edges that occurring at switching times

- **Advantages and extensions:**
  - formal semantics
  - Extension by hierarchies \(\rightsquigarrow\) abstraction on different levels
  - Extension by concurrency \(\rightsquigarrow\) modeling of multiple vehicles
Hybrid Automaton: resulting trajectories

\[ (q_1, q_1) \in E \]
\[ (q_2, q_1) \in E \]
\[ f: \dot{x} = f_{q_1}(x, u, t) \]
\[ i: g_{q_1}(x, u, t) \leq 0 \]
\[ (q_1, q_2) \in E \]
\[ f: \dot{x} = f_{q_2}(x, u, t) \]
\[ i: g_{q_2}(x, u, t) \leq 0 \]

\[ \dot{x} = f_{q_{ns-1}}() \]
\[ \dot{x} = f_{q_{ns}}() \]

initial condition \[ x(t_0) \]

flow condition \[ \dot{x} = f_{q_1}() \]

jump conditions and events

switching times

rules defining feasible sequences of switches
Example 1: robot soccer

- two controllable attackers (discrete tasks)
- one indirect controllable ball (switched motion dynamics)
- one reactive defender

Open issues:
- optimize attackers chances for a considered time horizon
- simultaneous task allocation and trajectory planning

⇒ long term goal: model predictive control
Example 1: basic hybrid automaton

**Game is running**

**Player 1 dribbles ball**
- $e$: kick(1)
  - $f$: $\dot{x}_1 = f_1, B(x_1, \dot{x}_1, u_1)$
  - $f$: $\dot{x}_2 = f_2(x_2, \dot{x}_2, u_2)$
  - $f$: $\dot{x}_B = f_{1, B}(x_1, \dot{x}_1, u_1)$
- $i$: $\text{dist}_{1, B} \leq \varepsilon_{\text{dribble}}$
- $i$: $g_1(x_1, u_1) \leq 0$
- $i$: $g_2(\dot{x}_2, u_2) \leq 0$

**Player 2 dribbles ball**
- $e$: kick(2)
  - $f$: $\dot{x}_1 = f_1(x_1, \dot{x}_1, u_1)$
  - $f$: $\dot{x}_2 = f_2(x_2, \dot{x}_2, u_2)$
  - $f$: $\dot{x}_B = f_{2, B}(x_2, \dot{x}_2, u_2)$
- $i$: $\text{dist}_{2, B} \leq \varepsilon_{\text{dribble}}$
- $i$: $g_1(\dot{x}_1, u_1) \leq 0$
- $i$: $g_2(\dot{x}_2, u_2) \leq 0$

**Ball free**
- $e$: catch(1)
  - $f$: $\dot{x}_1 = f_1(x_1, \dot{x}_1, u_1)$
  - $f$: $\dot{x}_2 = f_2(x_2, \dot{x}_2, u_2)$
  - $f$: $\dot{x}_B = f_B(x_b)$
- $i$: $\text{dist}_{1, B} > \varepsilon_{\text{dribble}}$
- $i$: $\text{dist}_{2, B} > \varepsilon_{\text{dribble}}$
- $i$: $g_1(\dot{x}_1, u_1) \leq 0$
- $i$: $g_2(\dot{x}_2, u_2) \leq 0$

**Ball in goal**
- $e$: goal
  - $f$: $\dot{x}_1 = 0$
  - $f$: $\dot{x}_2 = 0$
  - $f$: $\dot{x}_B = 0$
  - $f$: $\dot{x}_D = 0$

**Ball in goal**
- $j$: $|x_B| \geq x_{\text{field}}$
- $j$: $|y_B| \leq y_{\text{goal}}$
- $e$: catch(2)
  - $f$: $\dot{x}_1 = f_1(x_1, \dot{x}_1, u_1)$
  - $f$: $\dot{x}_2 = f_2(x_2, \dot{x}_2, u_2)$
  - $f$: $\dot{x}_B = f_{2, B}(x_2, \dot{x}_2, u_2)$
- $i$: $g_1(\dot{x}_1, u_1) \leq 0$
- $i$: $g_2(\dot{x}_2, u_2) \leq 0$
- $i$: $x_B \in \text{goal}$

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Modeling Cooperative Multi-Vehicle Systems

MILP-based Optimal Control of Cooperating Multi-Vehicle Systems

Towards On-line MPC

Summary and Outlook
**Control of multi-vehicle / multi-robot systems**

### non-linear hybrid optimal control:
- [Glocker; Barton & Lee, Rantzer...]
  - transformation into finite MINLP
  - needs initial guesses and bounds
  - maximum principle [Sussmann '99]
  - disjunctive programming [Oldenburg & Marquardt '07]
- high-dimensional mixed-integer NLP

### in practice:
- heuristic approaches to cover all expected situations (e.g., state machines [RoboCup])
- very task specific solutions
- assumptions are just roughly inspired by physics

### MILP-based optimal control [How ‘02, D’Andrea ’05; Bemporad, Stursberg, Engell ...]
- physics-based approximation
- numerical robustness and efficiency of MILP-solvers
- extension mpMILP for stable MPC
- globally optimal without need of guesses
From HOCP to MILP-based optimal control: linearization and hybridization

minimize
\[ \Phi = \sum_{s=1}^{n_s} (\varphi_s(x(t_s), u(t_s), t_s) + \int_{t_{s-1}}^{t_s} L(x(t), u(t)) \, dt) \]

subject to \(\forall i, j, q:\)
- (dynamics) \[ \dot{x}_i^q(t) = f_i^q(x_i^q(t), u_i^q(t)) \]
- \(0 \leq g_i^q(x_i^q(t), u_i^q(t)) \)
- \(0 \leq h_i^q(x_i^q(t)) \)

(mixed constr.) \(b_{q,i,j}(t) = 1 \Rightarrow C_q(x_i^q(t), x_j^q(t)) \leq 0 \)

(logical constraints) \[ L_q \leq \sum_{q=1}^{n_q} \sum_{s=1}^{n_s} \sum_{i=1}^{n_v} \pm b_{i,q}(t_s) \]

- polygonal approximation
- hybridization [e.g. Girard '07]
- big-M formulation [Wiliams '96]

minimize
\[ \sum_k \psi_k \cdot (x(k), u(k))^T \]

subject to \(\forall i, j, q, k:\)
- \(x_i^q(k+1) - x_i^q(k) = \Delta_t (A_i^q x_i^q(k) + B_i^q u_i^q(k)) \)
- \(C_{g,q}^i \leq G_{g,q}^i \cdot (x_i^q(k), u_i^q(k))^T \)
- \(C_{h,q}^i \leq G_{h,q}^i \cdot (x_i^q(k))^T \)
- \(b_{q,i,j}(k) \cdot M \geq \bar{C}_q(x_i^q(k), x_j^q(k))^T \)
- \(\tilde{L}_q \leq \sum_{q=1}^{n_q} \sum_{k=1}^{n_k} \sum_{i=1}^{n_v} \pm b_{i,q}(k) \)

- linearized difference equation
- fixed sampling time \(\Delta_t\)

non-linearity \(\sim\) more constraints and discrete structure
**Objective function** $\Phi(x(t), u(t), b(t))$

For fixed $[t_0, t_f]$ regarding

- state variables $x(k)$ (e.g. positions, distances),
- incidence of discrete states $b(k)$ (e.g. goal, ...),
- control variables $u(k)$ (e.g. energy input, acceleration).

- minimization of energy or “control efforts” for visiting all waypoints

$$\min \int_0^{t_f} u_x(t)^2 + u_y(t)^2 \, dt \quad \text{(HOCP)}$$

$$\leadsto \min \sum_k r_k \cdot \Delta t \quad \text{(lin. approx.)}$$
Numerical results: robot soccer

Optimal trajectories (x- and y-positions):

Respective optimal control:

- 12 timesteps: 1135 var., 1692 constr.
- Linearized dynamics:

\[ x_{k+1} = x_k + \Delta t \cdot v_k, \quad v_{k+1} = v_k + \Delta t \cdot u_k \]

- Solved with CPLEX in 9 sec.
- Computing time strongly depends on initial setting

PC with Intel Pentium M processor (1.86 GHz); 1GB RAM
Example 2: Monitoring with cooperating vehicles

- vehicle-specific motion dynamics (constraints on maximum velocities, controls...)
- certain areas have to be visited during the mission

Extensions:
- structured environment, obstacles
- required connectivity
- dense spatial distribution of many overlapping areas
- ...

⇒ (non-linear) optimal control subject to motion dynamics and heterogeneous, switched constraints
Numerical results: cooperative monitoring

optimal trajectories (x- and y-positions):

**linearized dynamics:**

\[ x_{k+1} = x_k + \Delta t \, v_k, \quad v_{k+1} = v_k + \Delta t \, u_k \]

**solved with CPLEX in 70.2 sec.**

**simultaneous** waypoint sequencing and trajectory optimization

PC with Intel Pentium M processor (1.86 GHz); 1GB RAM
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Summary and Outlook
Model predictive control for discrete time linear systems

- studied well for MLD and PWA systems [Bemporad, Morari, …]
- sophisticated implementation: Multi-Parametric Toolbox for Matlab [Kvasnica, Grieder, Baotić; ETHZ]

Example 3: Observing multiple targets with cooperating vehicles:

- multiple cooperating robots
- multiple (moving) targets
- a target is observed, if its position is within the robots observation radius $R$
- maximize amount and duration of observations

$\Rightarrow$ Discrete-valued optimal control problem subject to motion dynamics and switched constraints
Results:
MILP-model - MPC with online optimization

Example: 5 robots, 8 targets, 15 timesteps

\[
\sum_{k} \sum_{r} |u_{r,x}^{(k)}| + |u_{r,y}^{(k)}| = 703.46
\]

\[
\sum_{k} \sum_{r} |u_{r,x}^{(k)}| + |u_{r,y}^{(k)}| = 744.02
\]
Comparison
MILP-model - Online-MPC-controller

(thanks to J. Kuhn)

Example: 5 robots, 8 targets, 15 timesteps

solving MILP-model:
- full optimal control problem with 15 timesteps
- 10366 constraints
- 1936 variables
- computing time: 2.39h (!) (suboptimal solution after 60 s)

MPC with online optimization:¹
- 15 MPC calls with prediction horizon of 5 timesteps
- 3060 inequality constraints
- 706 Variables
- computing time: $15 \cdot 0.128 \text{s} = 1.92\text{s}$

¹Matlab MPT-toolbox using CPLEX; running on a PC (Intel Pentium 4 (3.00GHz), 1GB RAM)
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Summary and Outlook
Summary:

- physics-motivated modeling, analysis and optimization takes account for tight coupling of discrete states and continuous vehicle trajectories,
- MILP enables an efficient simultaneous optimization of cooperation and mobility,
- appropriate for motion dynamics with moderate non-linearity,
- first promising results with MPC-approaches,

Ongoing and related work:

- manage discrete structure in multi-parametric computation,
- extension to more realistic models (e.g. considering uncertainties),
- beneficial combinations with nonlinear approaches and CLP,
E. Asarin, T. Dang, and A. Girard.
Hybridization methods for the analysis of nonlinear systems.

A. Bemporad and M. Morari.
Control of systems integrating logic, dynamics, and constraints.

M. G. Earl and R. D’Andrea.
Iterative milp methods for vehicle control problems.

M. Glocker, C. Reinl, and O. von Stryk.
Optimal task allocation and dynamic trajectory planning for multi-vehicle systems using nonlinear hybrid optimal control.
In *Proc. 1st IFAC-Symposium on Multivehicle Systems*, pages 38–43, Salvador, Brazil, October 2-3 2006.

M. Kvasnica.
*Efficient software tools for control and analysis of hybrid systems.*

C. Reinl, M. Glocker, and O. von Stryk.
Optimalsteuerung kooperierender mehrfahrzeugsysteme.

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Optimal control of multi-vehicle systems under communication constraints using mixed-integer linear programming.
In *Proceedings of the. First International Conference on Robot Communication and Coordination (RoboComm)*, Athens, Greece, Oct. 15-17 2007. ICST.